Provisional Application for United States Patent

TITLE: Unified Differential Economics

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BACKGROUND

Capital markets and global economic behavior have long been studied as disjoint sets of events. In particular, within the broad fields of securities pricing, risk management, and financial economics, the daily (and intra-daily) phenomena that define these fields are analyzed by respective market participants under a plethora of methods. Such methods range in both complexity and fundamental understanding. For example, characterizing 10-year Treasury interest rate growth could by one market participant be estimated at as rudimentary of a level as elementary multiplication, while simultaneously another market participant characterizes such growth through exponential formulation, all while a third market participant applies the theory of Brownian Motion (via Monte Carlo simulation) to establish a sophisticated yield curve. In a separate objective, risk managers studying the shape of the yield curve apply numerous sensitivities, parallel shocks, and stresses to points Treasury curve in order to quantify (approximately) their capital at risk. Whether trading or risk managing, the methods used by market participants are only approximations to desired precisions, that serve the unique purposes of each participant.

- Periods of 1650-1688: Market participants relied on naïve assumption that there was a constant upward movement of stock prices.
- By late 1700's, trading options was introduced on London's exchange alley.
- In 17th and 18th centuries, trading on Amsterdam exchange has same features as modern derivative markets.
- By middle of 17th century, the Amsterdam Options Exchange becomes the first exchange trading financial derivative securities, including puts and calls with regular expiration dates.
- US stock option pricing in 1870s kept premium constant and adjusted the exercise price. The call prices *C* were the difference between exercise *K* and cash stock price *S* at any point in time *t*, where:

$$C(t) = \text{MAX}[\ 0, S(t) - K(t)\ ], \text{ where } t < T$$

- Bachelier c.1900: First financial engineer who utilized the theory of Brownian motion to explain the movement of the underlying stock. From that series of movements, he was ultimately able to derive an option price that considers all possible events on a Brownian distribution.
- Einstein c.1915: Formulated his covariant field equations which were based on Lagrangian notation which refined Newtonian physics in a curved space and was able to explain phenomena such as electromagnetism, gravity, and nuclear (weak and strong) forces.
- Harry Markowitz c.1950: Developed risk strategy and diversification as the optimal framework for asset allocation.
- Black Scholes (1973): Most important theoretical study of option pricing resulted in the Black Scholes formula, which expanded on the Brownian motion but also added the element of hedging and randomness to the third Taylor series expansion (via Ito's Lemma) to derive their equation which is still a cornerstone of option pricing today.

- The use of derivatives as risk management instruments arose during the 1970s, and expanded rapidly during the 1980s.
- International risk regulation began in the 1980s, and financial firms developed internal risk management models and capital calculation formulas to hedge against unanticipated risks and reduce regulatory capital.

As we have chronologically described above, the mathematical and economic frameworks governing the complex financial markets—particularly in the wake of: 1) derivatives trading, 2) optimal asset allocation, 3) intensified risk management, and 4) globalization—have been tailored made by participants to specifically handle their individual problems at hand, without consideration to how the methods are compatible (or incompatible) with other segments of the financial markets.

BRIEF SUMMARY OF THE INVENTION

> *"Our approach, called Unified Differential Economics, unifies the seemingly disjoint set of events occurring in the financial markets."*

This invention is a unified approach to securities pricing, asset allocation, risk management, and global financial economics.

Using the Treasury example posed above, our claim is that all such participants effectively shape the Treasury curve through their own trading and analysis of the 10-year Treasury, and the culmination of all such market activity exists within a geometric space that has its own set of principles and rules that define it. In particular, the field of physics was redefined when Einstein's unified framework of differential geometry and Riemannian mathematics began to fully explain the phenomena occurring within space and time. In the same fashion, economic theory has long been a victim of its own classical Newtonian physics which break down when applied across securities pricing, risk management, and financial economics. We have developed a unified framework to explain financial economic phenomena by transforming financial space into space-time manifolds which are governed by our tensor formulation under differential geometry and Riemann space.

Our approach, called *Unified Differential Economics*, unifies the seemingly disjoint set of events occurring in the four segments of the financial universe: securities pricing, asset allocation, risk management, and global economics.

As a byproduct to the unification, the Unified Differential Economics framework produces more accurate pricing models, risk management models, and trading strategies.

DETAILED DESCRIPTION AND BEST MODE OF IMPLEMENTATION

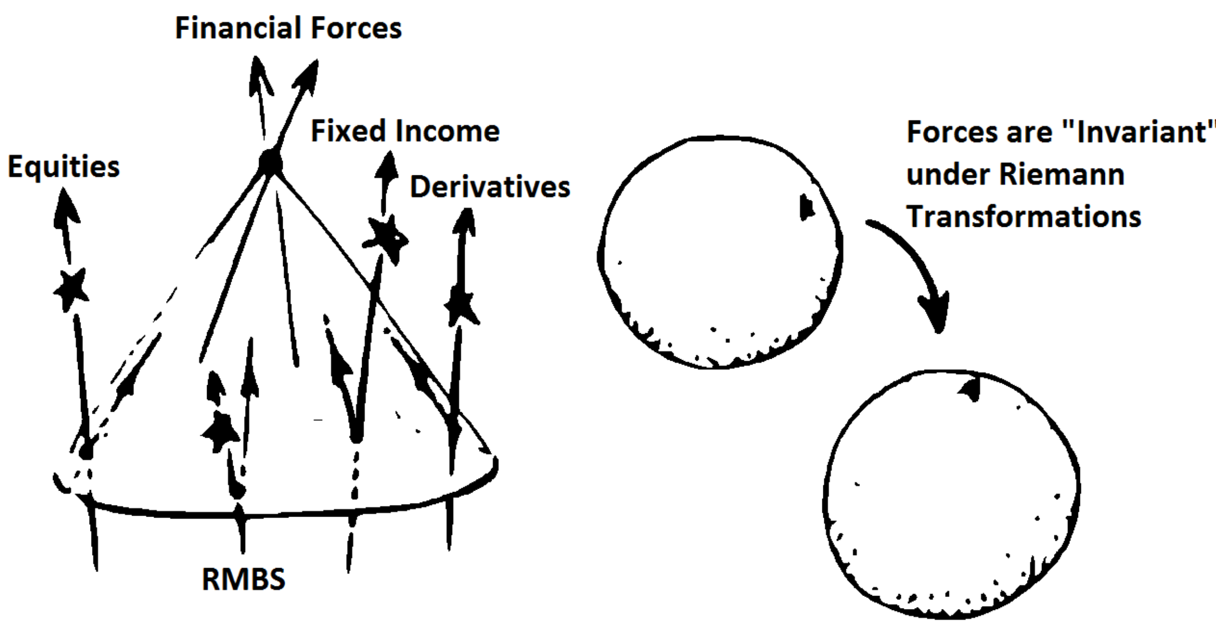
Unified Differential Economics unifies the mathematical and economic methods across the financial universe. Our equations and framework, which are adapted from Riemann's abstraction of mathematical space and Einstein's unification via relativistic physics, are seamlessly applied across the financial universe as made up by the four major segments: securities pricing, asset allocation, risk management, and global financial economics

We have developed a unified framework to explain financial economic phenomena by transforming financial space into space-time manifolds which are governed by our tensor formulation under differential geometry and Riemann space as follows:

Equation (1.1)

$$\frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x_{\alpha}} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} = -\kappa\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)$$

<u>Figure A1. Financial forces invariant transformation under Riemann space</u>



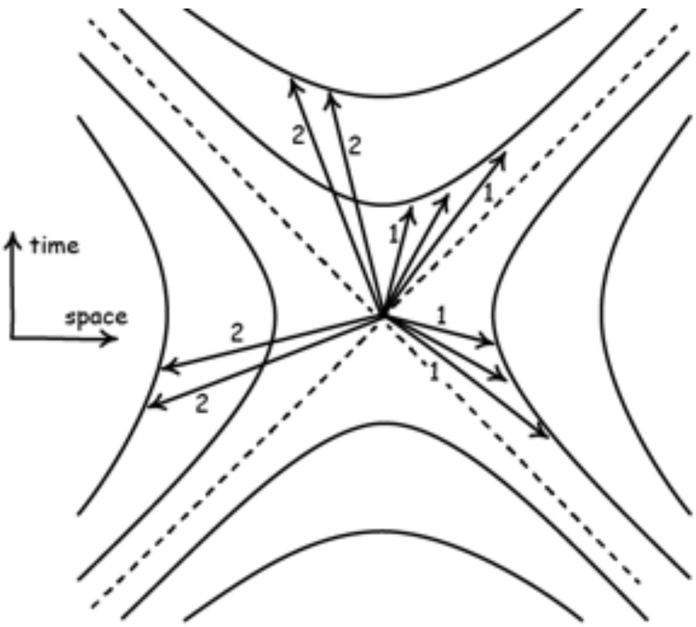
This equation is the foundation that unifies the financial forces below in *Figure A*.

Figure A. Disjoint forces in financial markets (partial list)

Securities Pricing	Risk Management and Financial Modeling
Equities	Interest rate term structure
Equity derivatives	Futures and forwards
Fixed income	Volatility surface
Credit derivatives	Inflation and deflation
Structured products	Global currency and foreign exchange
Asset backed securities	Tail event modeling
Sovereign and municipal debt	Commercial real estate
Alternative investments	Residential real estate

In one solution to our equation 1, the financial forces in *Figure A* by abstracting the geometric space into a 16 dimensional space-time warp in order to adjoin forces is opposing spin. The illustration of a highly curved space emerges when we express the differential to the 16th order, as shown in *Figure B*.

Figure B. *N*-dimensional manifold over Financial Forces with "opposing spin," where *N*=16



Because of the many forces occurring in the financial universe, a highly abstract and curved geometry is both a requirement and, as our empirical results prove, an advancement for the field of economics as it unifies the seemingly disjoint forces occurring within the financial markets.

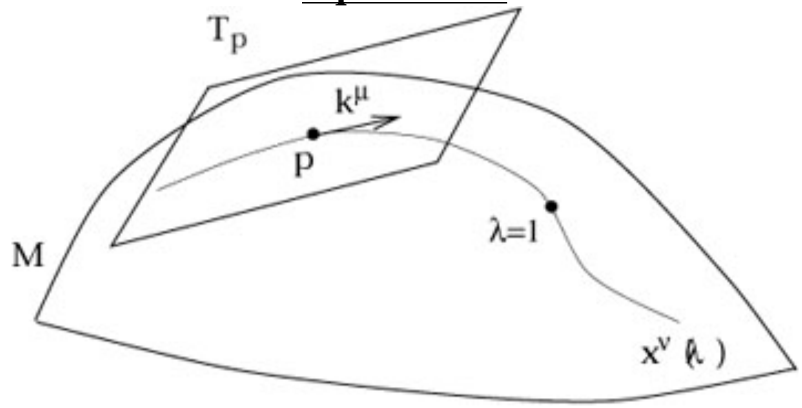
Shown above, *Figure A* lists sixteen (out of a much larger number) forces occurring in the financial universe, broken out between securities pricing and, more broadly, risk management and financial modeling. With respect to securities pricing, the modern methods fail to unite equities and fixed-income. As a byproduct to the unification of the financial forces, the Unified Differential Economics framework results in much more accurate pricing models, risk management models, and trading strategies.

Such forces are intrinsically united in the same way that both electromagnetic and gravitational forces are. All such forces exist within a single universe which has an over-arching mathematical framework describing it. We have united the securities pricing of the disjoint forces through differential geometry and Riemann space, where an N-dimensional Riemannian manifold is characterized by a second-order metric tensor $g_{\mu\nu}(x)$ which defines the differential metrical distance along any smooth curve in terms of the differential coordinate components according to Eq 1.2

Equation (1.2)

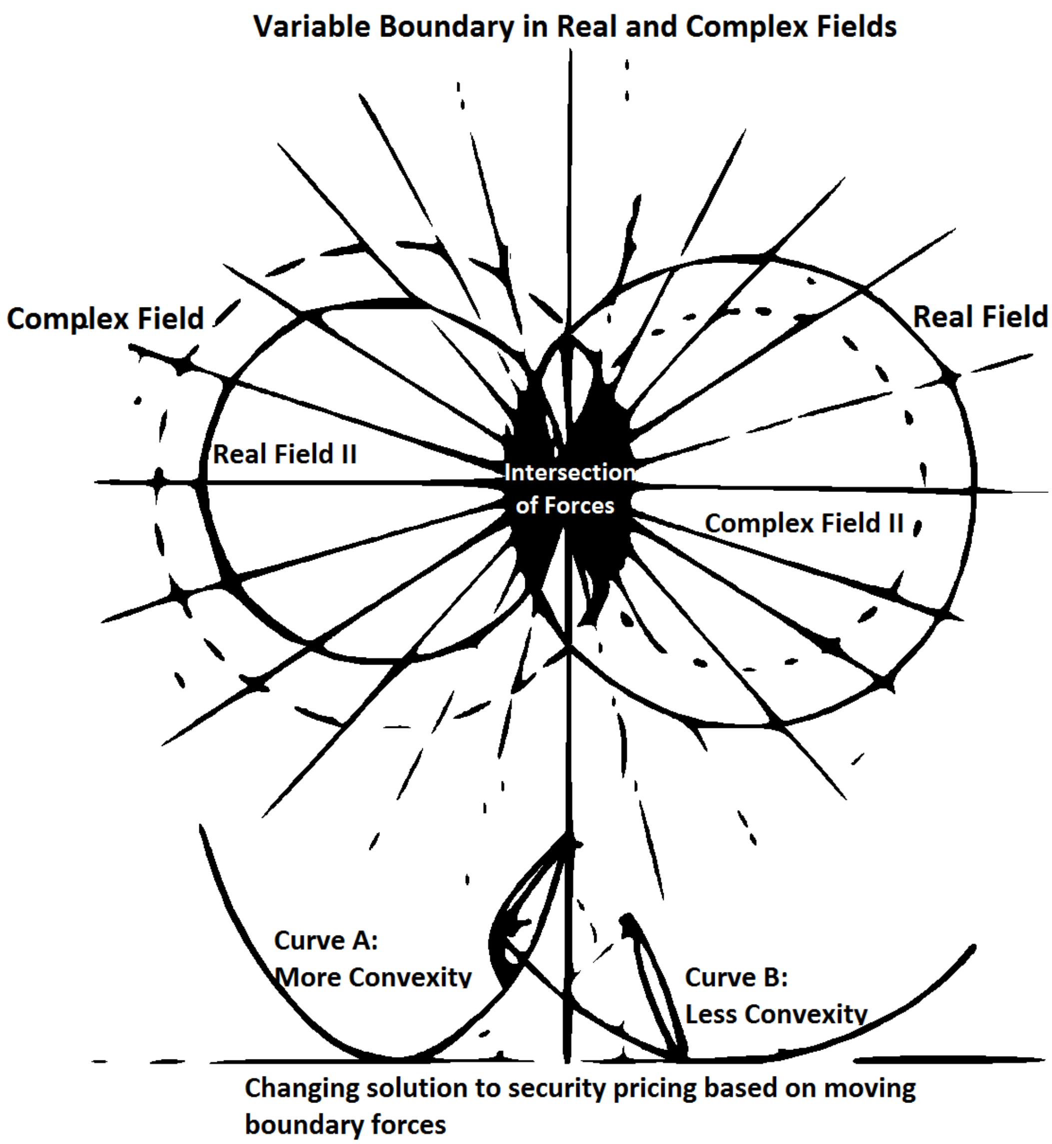
$$(ds)^2 = g_{\mu\nu}(x)\, dx^{\mu} dx^{\nu}$$

Figure B2. Solutions from Euclidean plane being transformed to solutions under Equation 1.2



We set the boundary conditions to be the forces in Figure A being both expressed outward and mutually inclusive. The resulting differential elliptic curve, represents each force, in show in in Figure C:

Figure C. Financial force invariance to changing (variable) boundary



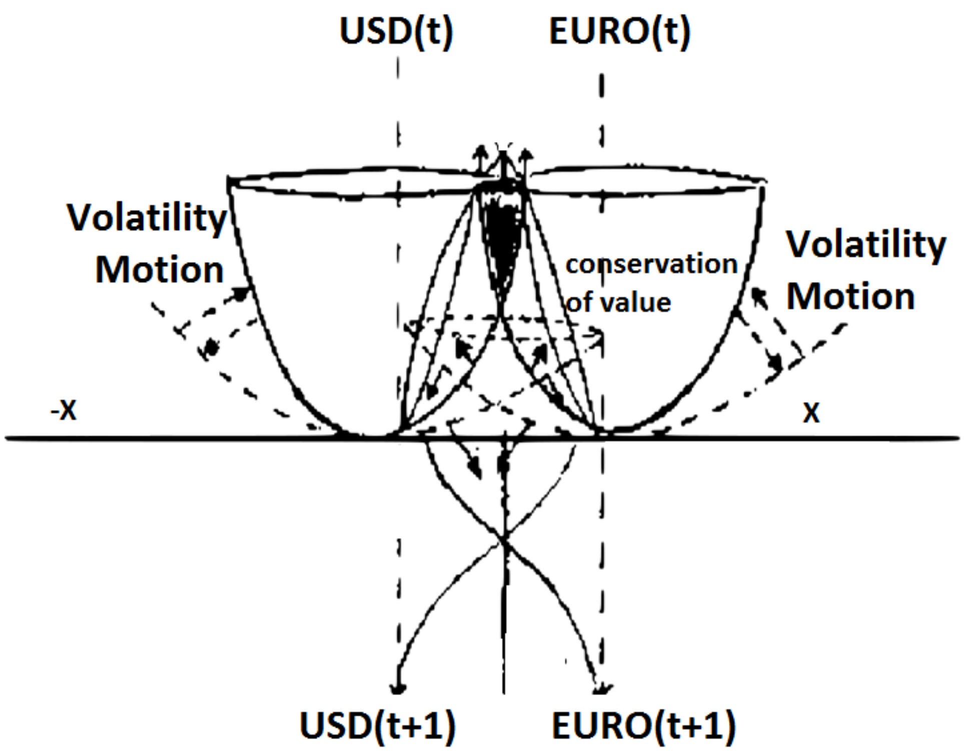
With a set of boundary conditions from Figure A, we define our pricing field as an interaction of the financial forces, as shown in Equation 2.

Equation (2)

$$-2\kappa t_{\sigma}^{\alpha} = g_{\sigma}^{\mu\nu} \frac{\partial L}{\partial g_{\alpha}^{\mu\nu}} - \delta_{\sigma}^{\alpha} L.$$

The above expresses the law of conservation of value (e.g. momentum) and volatility (e.g. energy) for the economic force field, as shown in Figure D.

Figure D. Conservation of currency value and volatility under the closed system



Using this conservation principle establishes the valuation of fixed-income securities under Eq 3.

Equation (3)

$$\frac{\partial}{\partial x_{\alpha}} \left(g^{\sigma\beta} \Gamma_{\mu\beta}^{\alpha}\right) = -\kappa \left[\left(t_{\mu}^{\sigma} + T_{\mu}^{\sigma}\right) - \frac{1}{2} \delta_{\mu}^{\alpha} (t + \mathbf{T})\right], \qquad \sqrt{-g} = 1.$$

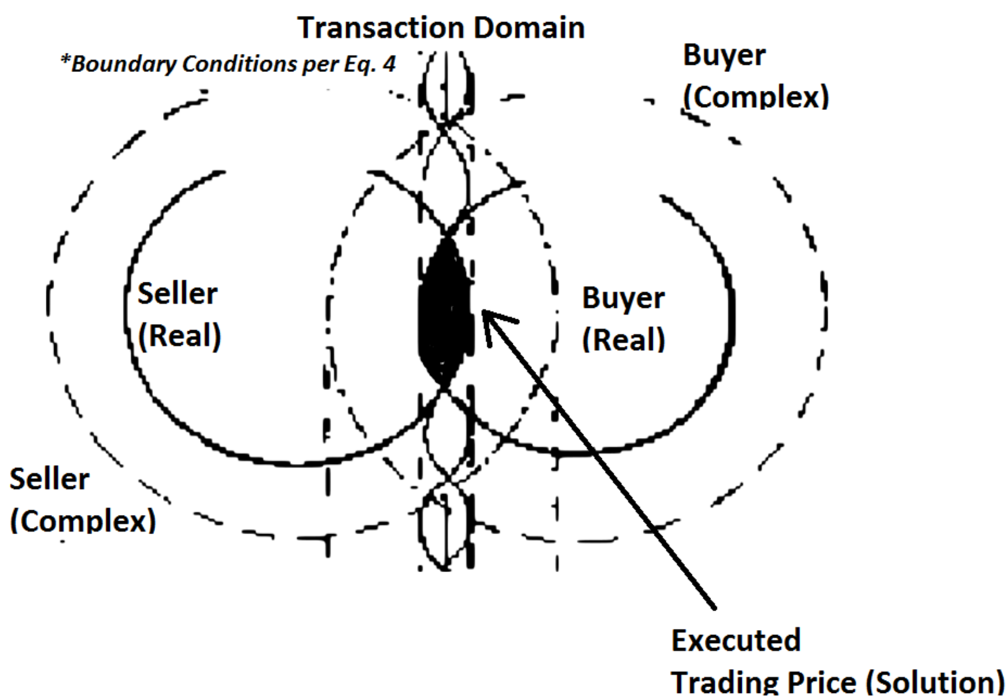
where we ensure the complex field exists by defining the coordinate condition $\sqrt{-g} = 1$.

Using a similar approach of Ito's Lemma as Scholes did, we can further the expansion of the above to a more generalized form of valuation which covers across both fixed-income and equities as shown in Eq 4:

Equation (4)

$$\frac{\partial \Gamma^{\alpha}_{\mu\nu}}{\partial x_{\alpha}} + \Gamma^{\alpha}_{\mu\beta}\Gamma^{\beta}_{\nu\alpha} = -\kappa\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)$$

***Figure E. Pricing of treasury bond under geometric space where buyer's force field (right) is symmetric to sellers force field (left).**



Under this framework described in Eq 4, we look at a case study where the valuation of Treasuries (with quarterly rate resets and semi-annual payments). In the Riemann manifold, the Treasury price which is invariant to all market rotations is given as the mutually inclusive set, V, in the cross section of the two countery-parties (showin in Figure E, where the Buyer-Right, Seller-Left)

The valuation of the above Treasury transaction follows from Eqns 3 and 4 as follows:

Equation (5)

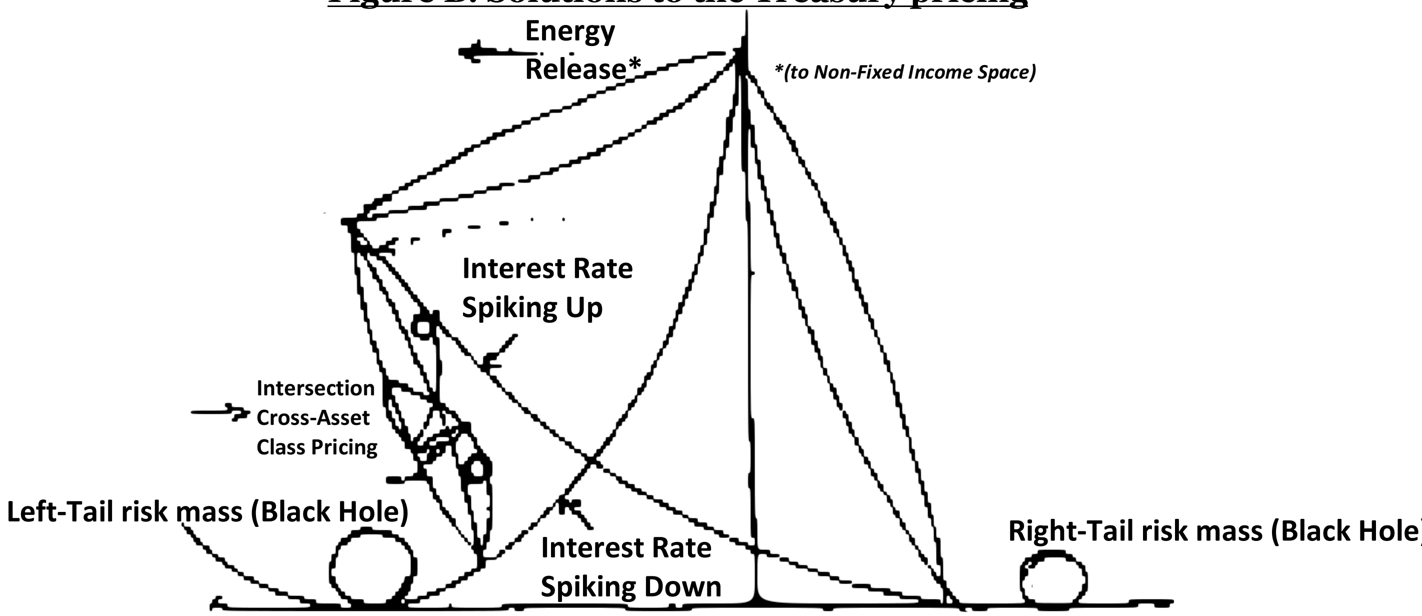
$$\frac{\delta V}{V} = \frac{1}{2}\left(\frac{\ddot{V}}{V}\right)(\delta t)^2 = -\frac{1}{4}\rho(\delta t)^2,$$

Equation (6)

$$
\begin{aligned}
\delta V_S &= \delta V_P \\
&= \left(\frac{\delta V}{V}\right) V_P \\
&= -\frac{1}{4}\rho(\delta t)^2\, V_P \\
&= -\frac{1}{4}M(\delta t)^2.
\end{aligned}
$$

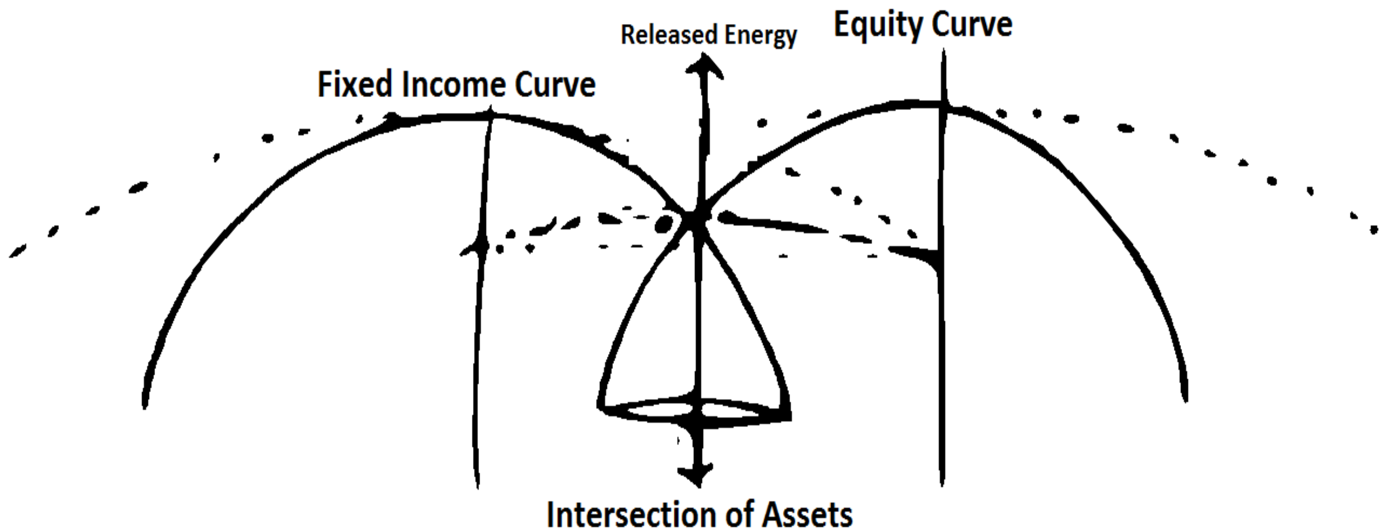
The solutions of Treasury curves under the above equations are illustrated in *Figure B*, where *R* is the Rate curve and *t* is the tenor (time of the term structure):

Figure B. Solutions to the Treasury pricing



The curvature is also what allows us to transform from fixed-income to equity space. See Figure F below which is a basic illustration of how the differential economic theory can allow pricing across the fixed-income-equity curve. Equations 7 and 8 describe the change in Rate structure of Treasuries when transformed into equity space.

Figure F. Basic example of how to transform Riemannian space from fixed-income to equity under Lie Group invariance.



Equation (7)

$$\lim_{\epsilon \to 0} \frac{w_2 - w_1}{\epsilon^2} = R(u,v)w$$

Equation (8)

$$R(u,v)w = R^{\alpha}_{\beta\gamma\delta} u^{\beta} v^{\gamma} w^{\delta},$$

Through the progression of physics, our understanding of the universe was redefined when Einstein's unified framework of Relativity told through differential geometry and Riemannian mathematics began to fully explain the phenomena occurring within space and time, phenomena which could not be explained by Newtonian physics under Euclidean space.

In the same fashion, economic theory has long been constrained through classical Euclidean frameworks which break down when applied across securities pricing, risk management, and financial economics. This is precisely why options traders realized that the volatility curve described by the Black Scholes equation fails at the tails of the distribution, due to the log-normal assumption which is in violation of how prices fluctuate in reality, and as a result caused extreme financial losses to large trading institutions such as LTCM, Lehman Brothers, and Bear Stearns.

Under a Unified Differential Economics framework, such losses would have been prevented and numerous cross-asset class pricing could have given way to hedges across the global financial markets. Our innovative and revolutionary framework of the behavioral (e.g. greed, fear, uncertainty, doubt), mathematical, and physical foundations of finance unifies the seemingly disjoint set of events occurring in the four segments of the financial universe: securities pricing, asset allocation, risk management, and global economics.

As a byproduct to the unification, the Unified Differential Economics framework produces more accurate pricing models, risk management models, and trading strategies.

CLAIMS

Our claim is that Unified Differential Economics unifies the mathematical and economic methods across the financial universe.

A Unified Differential Economics mathematical framework that unifies the mathematical and economic methods across the financial universe. Our equations and framework, which are adapted from Riemann's abstraction of mathematical space and Einstein's unification via relativistic physics, are seamlessly applied across the financial universe as made up by the four major segments: securities pricing, asset allocation, risk management, and global financial economics.

The Unified Differential Economics framework produces more accurate pricing models, risk management models, and trading strategies.

ABSTRACT:

The Unified Differential Economics framework unifies the mathematical and economic methods across the financial universe. Our equations and framework, which are adapted from Riemann's abstraction of mathematical space and Einstein's unification via relativistic physics, are seamlessly applied across the financial universe as made up by the four major segments: securities pricing, asset allocation, risk management, and global financial economics.